THEORY OF $B \to X_s \ell^+ \ell^-$

[testing the SM with $b \to s \ell^+ \ell^-$]

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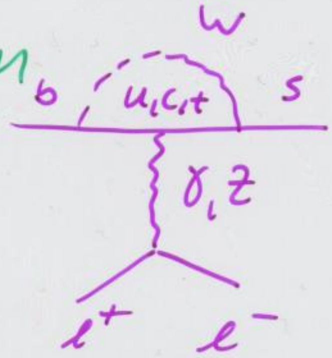
RINGBERG WORKSHOP 1.5.2003

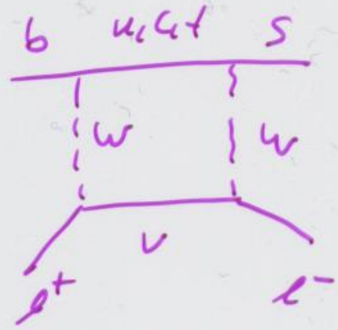
OUTLINE

- INTRODUCTION TO H_{eff}
- EXP. STATUS & THEORY PRECISION IN $Br(B \to X_s \ell^+ \ell^-)$ $\ell = e, \mu$ @ NNLO
- DISKUSSION THEORY UNCERTAINTIES
- MODEL INDEPENDENT ANALYSIS

WHAT'S NEXT AFTER $b \to s\gamma$?

$b \to s\, \ell^+ \ell^-$ in SM





$$H_{eff} = \frac{4 G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i O_i$$

dipole : $\underline{O_7} \sim m_b \bar{s}_L \sigma_{\mu\nu} b_R F^{\mu\nu}$; $O_8 \sim m_b \bar{s}_L \sigma_{\mu\nu} b_R G^{\mu\nu}$

4-Fermi : $\underline{O_9} \sim \bar{s}_L \gamma_\mu b_L \bar{\ell} \gamma^\mu \ell$; $\underline{O_{10}} \sim \bar{s}_L \gamma_\mu b_L \bar{\ell} \gamma^\mu \gamma_5 \ell$

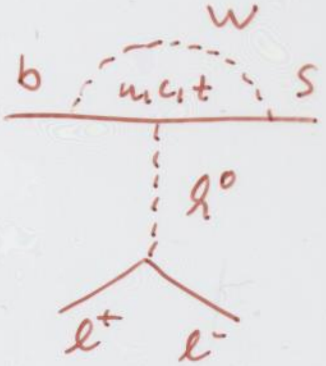
Z-penguin and box also in $b \to s \nu \bar{\nu}$, $b \to s q \bar{q}$

<u>Beyond the SM</u> $\quad C_i \to C_i^{SM} + C_i^{NP}$

AND/OR $\quad$ new operators

e.g. helicity flipped $O_i' = O_i$ with $L \leftrightarrow R$ in $\bar{s} \Gamma b$

in SM (and MFV): $C_i' = \frac{m_s}{m_b} C_i$



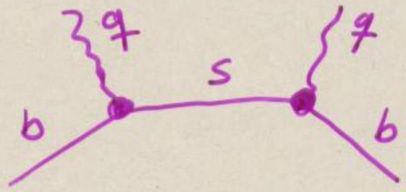
scalar / pseudoscalar

$O_S \sim \bar{s}_L b_R \bar{\ell} \ell$

$O_P \sim \bar{s}_L b_R \bar{\ell} \gamma_5 \ell$

$C_{S/P}^{SM} \sim \frac{m_\ell m_b}{m_W^2}$ very small even for τ

INCLUSIVE $b \to s\, \ell^+\ell^-$ @ NNLO

$d\Gamma \sim \mathrm{Im}\Big($  $\Big)$

[SEE TALK by M. LUKE on $b \to u\, \ell\nu$]

$$\frac{d\Gamma(B\to X_s \ell^+\ell^-)}{d\hat{s}} = \left(\frac{\alpha}{4\pi}\right)^2 \frac{G_F^2 m_b^5 |V_{tb}V_{ts}^*|^2}{48\pi^3}(1-\hat{s})^2$$

$$* \Big[(1+2\hat{s})\left(|C_9^{eff}|^2 + |C_{10}^{eff}|^2\right) f_1(\hat{s}) + 4\left(1+\frac{2}{\hat{s}}\right)|C_7^{eff}|^2 f_2(\hat{s}) + 12\,\mathrm{Re}\left(C_7^{eff} C_9^{eff*}\right) f_3(\hat{s}) + f_c(\hat{s})\Big]$$

$$f_{1,2,3}(\hat{s}) = 1 + \frac{\lambda_1}{2m_b^2} + \frac{O(\lambda_2)}{2m_b^2} \;;\; f_c(\hat{s}) \sim \frac{\lambda_2}{m_c^2}$$

hqet correct.

$$C_i^{eff} = \left[1 + \frac{\alpha_s}{\pi}\omega_i(\hat{s})\right]\left[C_i + \dots\right] + \frac{\alpha_s}{4\pi} C_j F_{ij}(\hat{s})$$

F_{ij} : virtual corrections

ω_i : α_s-corrections of $\langle O_i \rangle$ including bremsstrahlung

POWER COUNTING

$$C_9 \sim \frac{1}{\alpha_s} + 1 + \alpha_s + \dots$$

$$C_{7,10} \sim \phi + 1 + \alpha_s + \dots$$

LO NLO NNLO

NLO ($b\to s\gamma$) $\hat{=}$ NNLO ($b \to s\ell^+\ell^-$)

$b \to s\ell^+\ell^-$ @NNLO SEE TALK by T. HURTH

$b \to s\ell^+\ell^-$ status

⋆⋆⋆ 2001 first observation of exclusive decay ⋆⋆⋆

$\mathcal{B}(B \to K\ell^+\ell^-)_{SM} = 0.35 \pm 0.12 \cdot 10^{-6}$ NNLO hep-ph/0112300

$\mathcal{B}(B \to K\ell^+\ell^-) = 0.58^{+0.17}_{-0.15} \pm 0.06 \cdot 10^{-6}$ Belle prelim. ICHEP 2002

$\mathcal{B}(B \to K\ell^+\ell^-) = 0.78^{+0.24+0.11}_{-0.20-0.18} \cdot 10^{-6}$ BaBar prelim. ICHEP 2002

$\mathcal{B}(B \to K^*\mu^+\mu^-)_{SM} = 1.19 \pm 0.39 \cdot 10^{-6}$ NNLO hep-ph/0112300

$\mathcal{B}(B \to K^*e^+e^-)_{SM} = 1.58 \pm 0.49 \cdot 10^{-6}$ NNLO hep-ph/0112300

$\mathcal{B}(B \to K^*\ell^+\ell^-) < 1.4 \cdot 10^{-6}$ @ 90%C.L. Belle prelim. ICHEP 2002

$\mathcal{B}(B \to K^*\ell^+\ell^-) < 3.0 \cdot 10^{-6}$ @ 90%C.L. BaBar prelim. ICHEP 2002

inclusive mode	Br_{exp} Belle prelim'02	signif.	Br_{SM} NNLO hep-ph/0112300
$B \to X_s\mu^+\mu^-$	$7.9 \pm 2.1^{+2.0}_{-1.5} \cdot 10^{-6}$	4.7σ	$4.15 \pm 0.70 \cdot 10^{-6}$
$B \to X_s e^+e^-$	$5.0 \pm 2.3^{+1.2}_{-1.1} \cdot 10^{-6}$	3.4σ	$6.89 \pm 1.01 \cdot 10^{-6}$
$B \to X_s\ell^+\ell^-$	$6.1 \pm 1.4^{+1.3}_{-1.1} \cdot 10^{-6}$	5.4σ	

ERROR BUDGET & PROSPECTS

$\frac{m_b}{2} < \mu < 2 m_b \quad m_t^{pole} = (173 \pm 5)\,GeV \quad \frac{m_c}{m_b} = 0.29 \pm 0.04$

$$Br(B \to X_s e^+ e^-)^{NNLO}_{SM} = 6.89 \pm 0.37 \pm 0.25 \pm 0.91 * 10^{-6}$$

$$\delta Br_{e^+e^-} = \pm 15\%$$

$$Br(B \to X_s \mu^+ \mu^-)^{NNLO}_{SM} = 4.15 \pm 0.27 \pm 0.21 \pm 0.62 * 10^{-6}$$

$$\delta Br_{\mu^+\mu^-} = \pm 17\%$$

NLO → NNLO:
- μ-dependence decreased from 13% → 6.5%
- Branching ratios decreased by 12% (e^+e^-) 20% ($\mu^+\mu^-$)

PROSPECTS AT B-FACTORIES [G. EIGEN hep-ex/0112041]

	SUMMER 2002	2005	2010	
$\mathcal{L}\,[fb^{-1}/y]$	100	500	1000	
yield $b \to s e^+e^- (\mu^+\mu^-)$	73 (57)	365 (280)	728 (565)	
σ_{STAT} [%]	17 (19)	7 (9)	5 (6)	
σ_{SYS} [%]	10 (17)	7 (12)	6 (10)	

CHARM MASS DEPENDENCE

2 sources: in rare decay and from normalization

$$Br(b \to s \ell^+ \ell^-) = B_{se} \frac{\Gamma(b \to s \ell^+ \ell^-)}{\Gamma(b \to c \ell \nu)} \qquad (1)$$

study parametric dependence in rates $z = \frac{m_c}{m_b}$

with $\Delta(z) = \frac{\Gamma(z) - \Gamma(0.29)}{\Gamma(0.29)} \approx \frac{(0.29 - z)}{0.02} \cdot \varepsilon$

	ε
$b \to s \nu \bar{\nu}$	0
$b \to s \ell^+ \ell^-$	1 %
$b \to s \gamma$	3 %
$b \to c \ell \nu$	8 %

biggest effect from normalization

m_c^{POLE} vs $\overline{m}_c$? $\to$ FIG $\boxed{\begin{array}{c}0.29 \pm 0.04 \\ \text{CONSERVATIVE}\end{array}}$

ALTERNATIVES TO (1): [BECHER, NEUBERT for $b \to s \gamma$]

- normalize to lifetime

  $Br(b \to s \ell^+ \ell^-) \sim \tau(B)\, m_b^5\, |V_{tb} V_{ts}^*|^2$

  from m_b: $\delta\Gamma \gtrsim 5 \cdot \delta m_b = 10\%$

  for $m_b^{1S} = 4.7 \pm 0.1$ GeV

- a la GAMBINO, MISIAK for $b \to s \gamma$

$$Br(b \to s \ell^+ \ell^-) = \frac{\Gamma(b \to s \ell^+ \ell^-)}{\Gamma(b \to u \ell \nu)} \cdot \underbrace{\frac{\Gamma(b \to u \ell \nu)}{\Gamma(b \to c \ell \nu)}}_{\equiv R \cdot \left|\frac{V_{ub}}{V_{cb}}\right|^2}$$

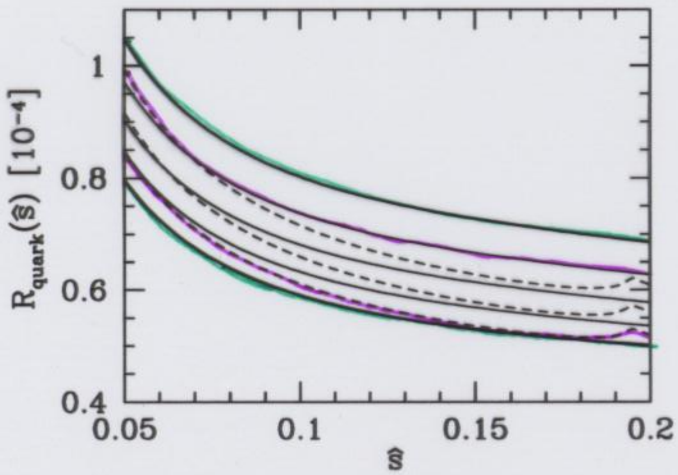
error on R small

$\varepsilon_R \simeq 1\%\,(\text{pert.}) + 2\%\,(\lambda_1) + 2\%\,(m_b^{1S})$ GAMBINO MISIAK

charm mass effects in $b \to s\ell^+\ell^-$

plot of branching ratio of $b \to s\ell^+\ell^-$ from hep-ph/0204341

solid:$m_c/m_b = 0.33$ (upper most) $m_c/m_b = 0.31, 0.29, 0.27, 0.25$ in both semileptonic decays; dashed: pole mass $m_c/m_b = 0.31, 0.29, 0.27$ in Γ_{sl},$\bar{m}_c/m_b = 0.22$ in rare decay $m_c/m_b = 0.29 \pm 0.04$ conservative



approximate — range
by — range

with $\frac{m_c}{m_b} = 0.29 \pm 0.04$

exclusive $B \to K, K^*\ell^+\ell^-$ decays: normalize to life time $\delta\tau(B) = 1\%$

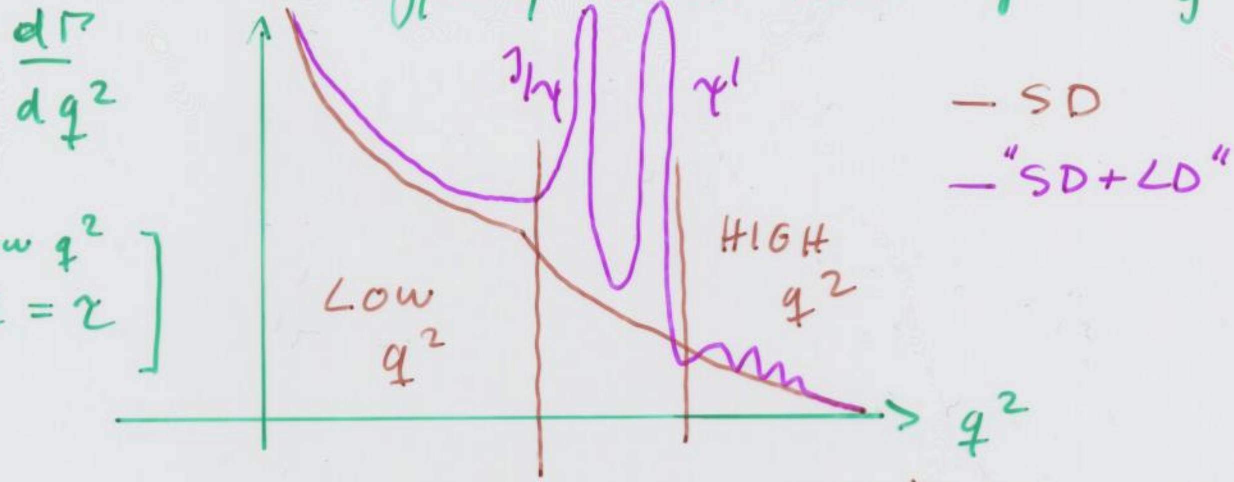
normalize inclusive to $\tau(B)$? $\Gamma(b \to s\ell^+\ell^-) \sim m_b^5 |V_{tb}V_{ts}^*|^2$

from m_b: $\delta\Gamma \simeq 5\delta m_b \simeq 10\%$ for $m_b^{1S} = 4.7 \pm 0.1$GeV will improve

CHARMONIUM EFFECTS

BGD from $b \to s(\bar{c}c) \to s\,\ell^+\ell^-$ peaks near
$q^2 \approx m^2_{J/\psi}, m^2_{\psi'}, m^2_{\psi''}, \ldots$

non perturbative effect, include Breit-Wigner in C_9^{eff}



[no low q^2 for $\ell = \tau$]

double counting? USE DATA [KRÜGER SEHGAL]

$$R^{c\bar{c}}_{had} = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}\Bigg|_{c\bar{c}} = \underbrace{R^{c\bar{c}}_{cont}}_{\text{fit to data}} + \underbrace{R^{c\bar{c}}_{RESON}}_{\text{Breit-Wigner}}$$

$$C_9^{eff} = C_9^{\xi} + (3C_1 + C_2)\cdot g(\text{charm loop}) + \text{Penguins} + \xi\text{-dependent constant}$$

$$\mathrm{Im}(g) = \frac{\pi}{3} R^{c\bar{c}}_{had}$$

ARE DATA GOOD ENOUGH? $\longrightarrow$ FIG

$\varepsilon R^{c\bar{c}}_{had} \gtrsim 15\%$ $\longrightarrow$ FIG YES!

$\longrightarrow$ FIG

[method supported by $1/m_c$ expansion
OK below charm threshold
rely on factorization]

for $4m_\mu^2 \le q^2 < 6\,\text{GeV}^2$ $+2.1\%$ correction w.r.t. SD

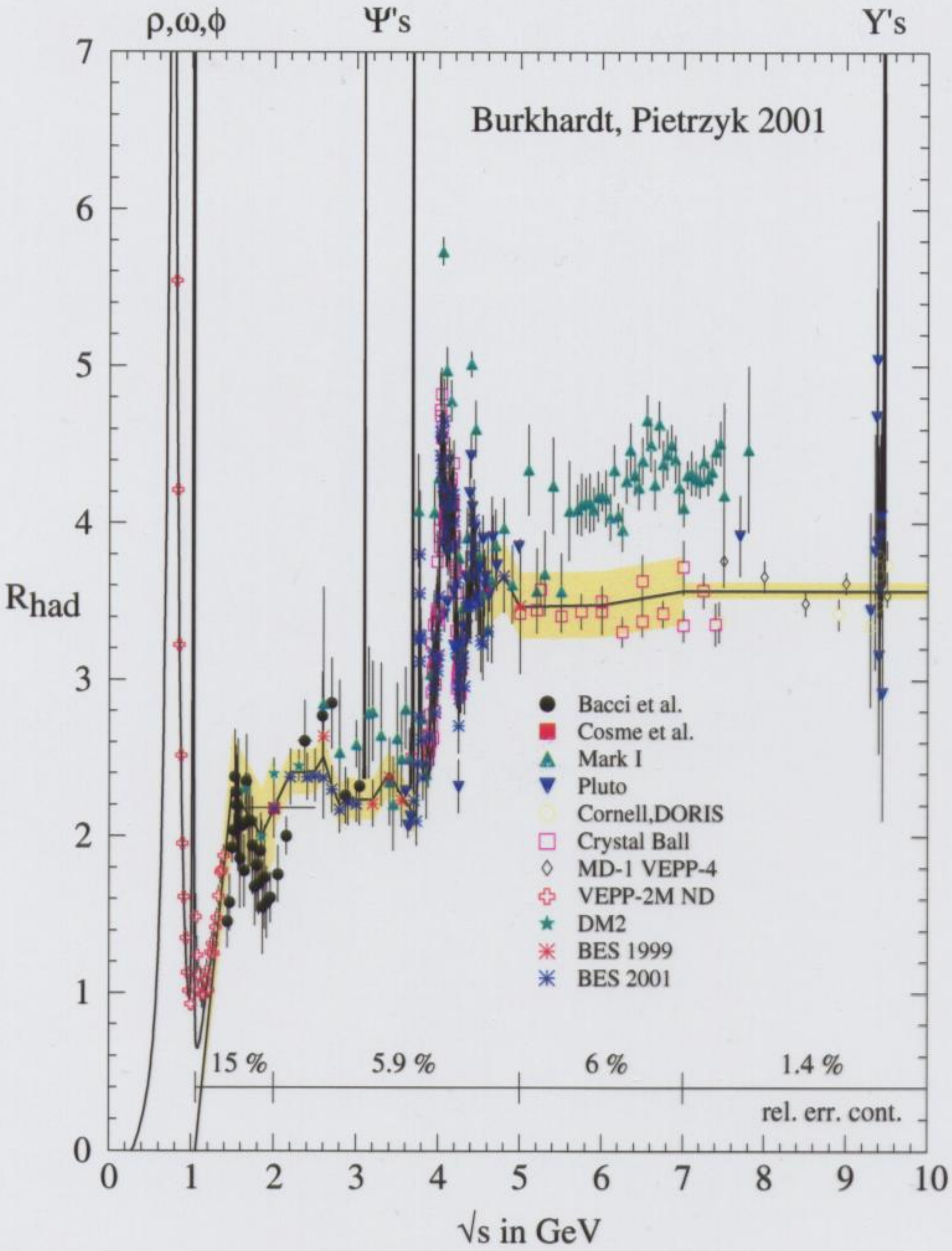
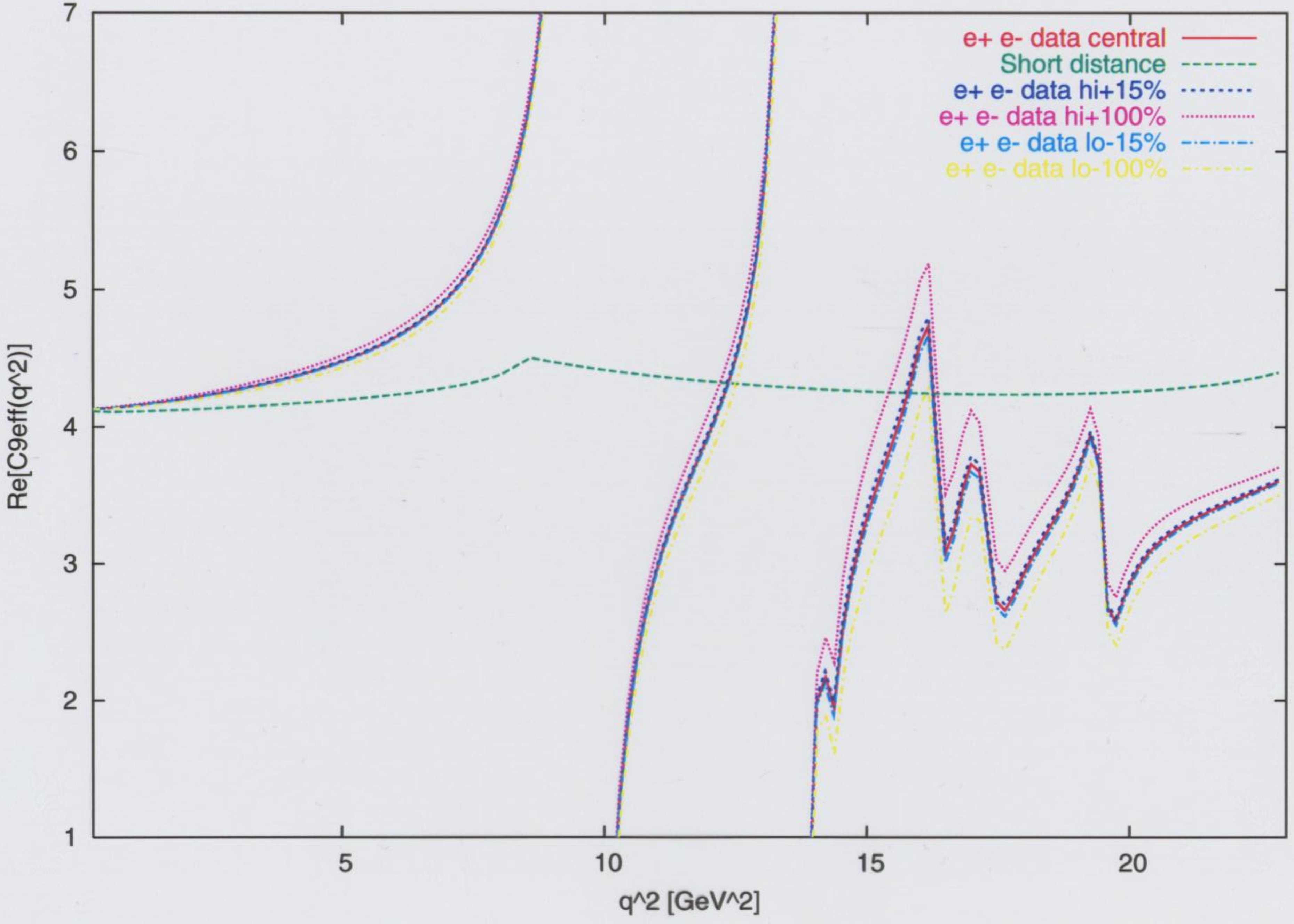
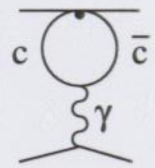


Fig. 1. R_{had} including resonances. Measurements are shown with statistical errors. In addition there are overall systematic errors (up to 20% in case of Mark I). The relative uncertainty assigned to our parametrization is shown as band and given with numbers at the bottom.

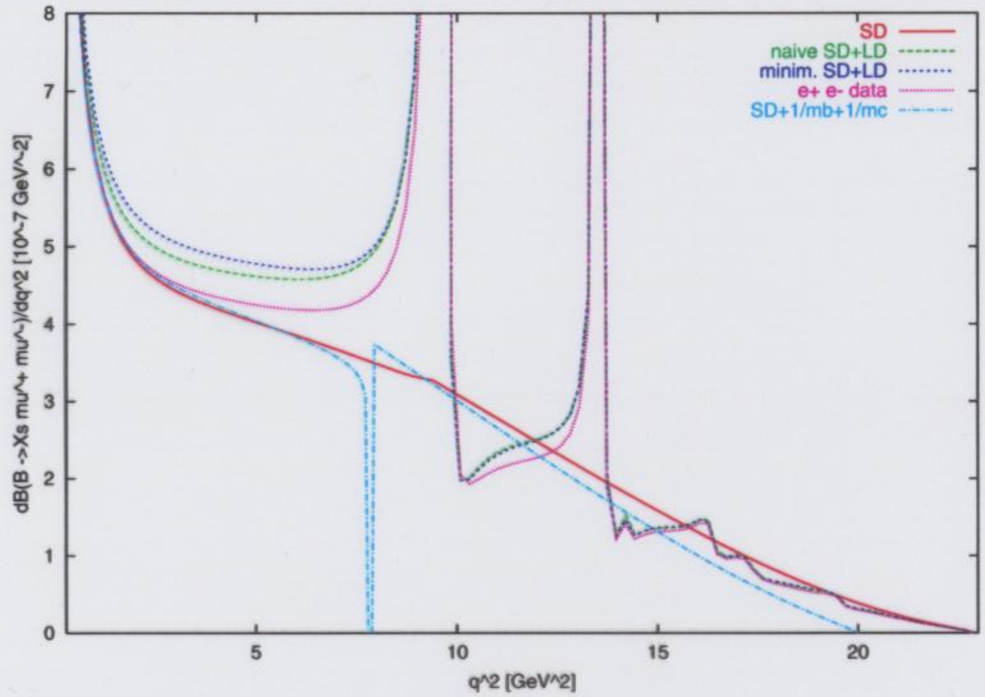

e+ e- data central
Short distance
e+ e- data hi+15%
e+ e- data hi+100%
e+ e- data lo-15%
e+ e- data lo-100%
Re[C9eff(q^2)]
7
6
5
4
3
2
1
5
10
15
20
q^2 [GeV^2]

charmonium effects in $b \to s\ell^+\ell^-$ decays



BGD from $b \to s(c\bar{c}) \to s\ell^+\ell^-$ peaks near $q^2 \sim m^2_{J/\Psi}, m^2_{\Psi'}, m^2_{\Psi''}\ldots$

not captured by perturbation theory, add Breit-Wigner



cuts in q^2 required; double counting? leakage away from resonances?

$e^+e^- \to hadrons$ data (pink) supported by $1/m_c$ expansion (light blue)

ok below charm threshold – rely on factorization

HADRONIC INVARIANT MASS CUTS

EXPERIMENTAL CUTS

similar to inclusive $b \to s\gamma$, $b \to u \ell \nu$:

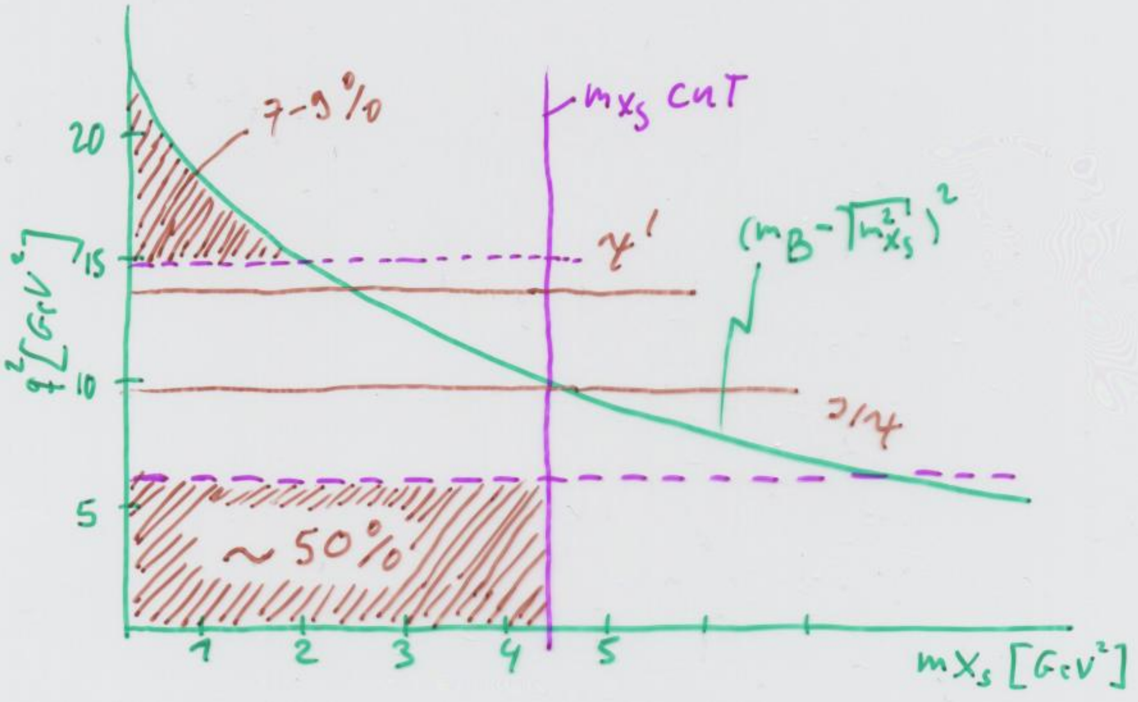
OPE valid ? Fermimotion effects

Belle '02 uses $m_{X_s} \leq 2.1$ GeV to suppress

BGD from $b \to c\, \ell^- \bar{\nu}$, $\;\llcorner\!\to s\, \ell^+ \nu$ $\Big\} = s\,\ell^+\ell^- +$ missing energy

$m_{X_s}^{CUT} \sim \sqrt{\Lambda \cdot m_b}$ shape function + resonance region

[this m_{X_s} CUT in $b \to s\gamma$ implies $E_\gamma > 2.1$ GeV]



low q^2 region : ⊕ $C_7 - C_9$ interference ⊕ $(c\bar{c})$ BGD

⊖ not optimal cut ⊕ many events

m_{X_s}- spectrum : bremsstrahlung & Fermimotion

fit parameters to E_γ-spectrum in $b \to s\gamma$ → FIG

subleading effect : boost b-cms to B-cms

NOT TOO BAD ON efficiency $\left[\int_{m_K}^{m_{X_s} cut} dx_s \frac{dBr}{dx_s} \right] / Br = 93 \pm 4\%$

HADRONIC INVARIANT MASS CUTS

EXPERIMENTAL CUTS

similar to inclusive $b \to s\gamma$, $b \to u \ell \nu$:

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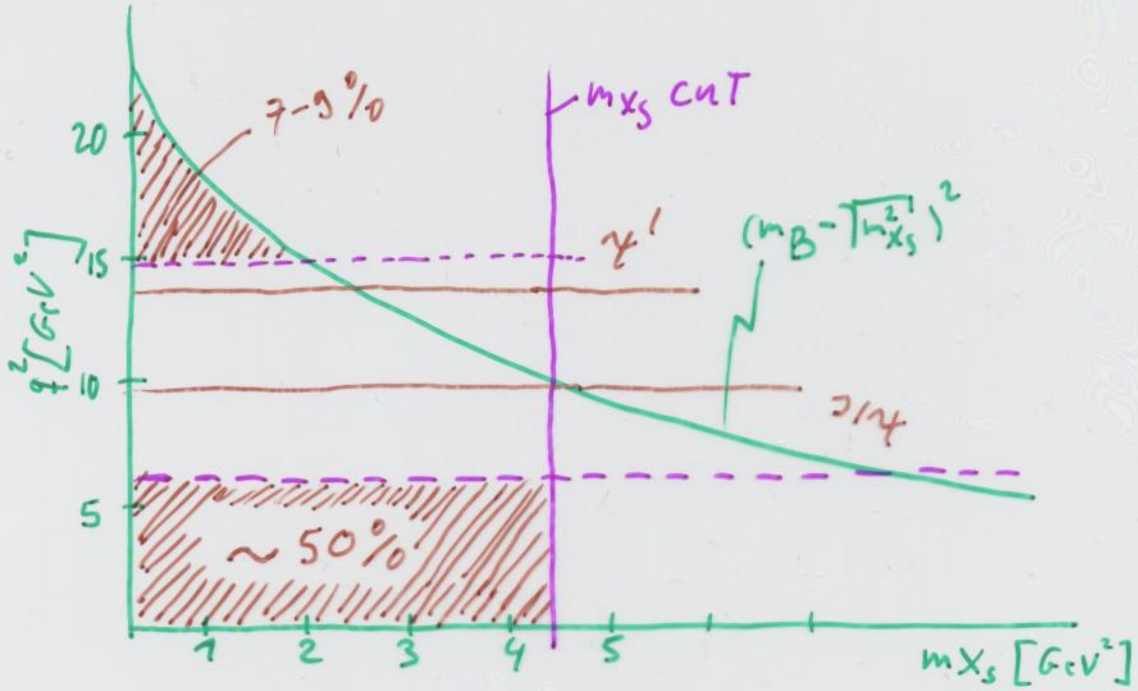
$m_{X_s}^{CUT} \sim \sqrt{1 \cdot m_b}$ shape function + resonance region

[this $m_{X_s}^{CUT}$ in $b \to s\gamma$ implies $E_\gamma > 2.1$ GeV]



low q^2 region : ⊕ $C_7 - C_9$ interference ⊕ $(c\bar{c})$ BGD

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m_{X_s} - spectrum : bremsstrahlung & Fermi motion

fit parameters to E_γ-spectrum in $b \to s\gamma$ → FIG

subleading effect : boost b-cms to B-cms

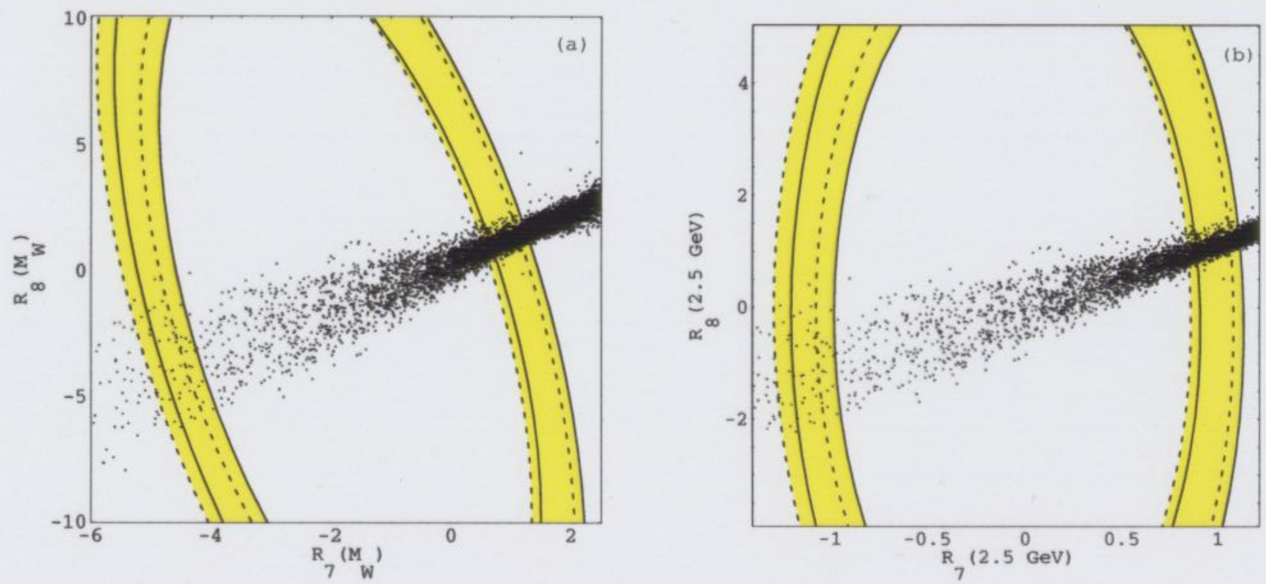
NOT TOO BAD ON Efficiency $\left[\int_{m_K}^{m_{X_s}cut} dX_s \frac{dBr}{dX_s}\right] / Br = 93 \pm 4\%$

constraints from $b \to s\gamma$ branching ratio

model independent analysis with $\mathcal{H}_{eff}$ C_7, C_8 eff. $bs\gamma, bsg$ couplings

$O_7 \propto \bar{s}_L \sigma_{\mu\nu} b_R F^{\mu\nu}$, $O_8 \propto \bar{s}_L \sigma_{\mu\nu} b_R G^{\mu\nu}$ $\mathcal{B}(b \to s\gamma)_{LO} \sim |C_7(m_b)|^2$

ratios $R(\mu) \equiv \frac{C^{SM}(\mu)+C^{NP}(\mu)}{C^{SM}(\mu)}$ NLO update @90%C.L. hep-ph/0112300



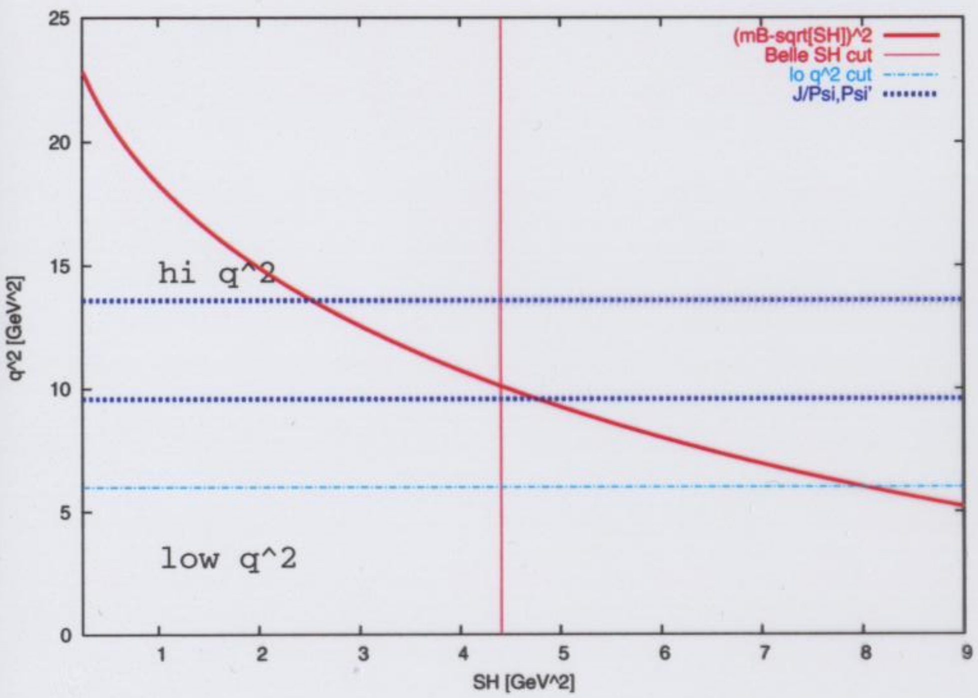
theory errors μ and charm mass solid:pole, dashed:$\overline{\text{MS}}$ prospects: 2005

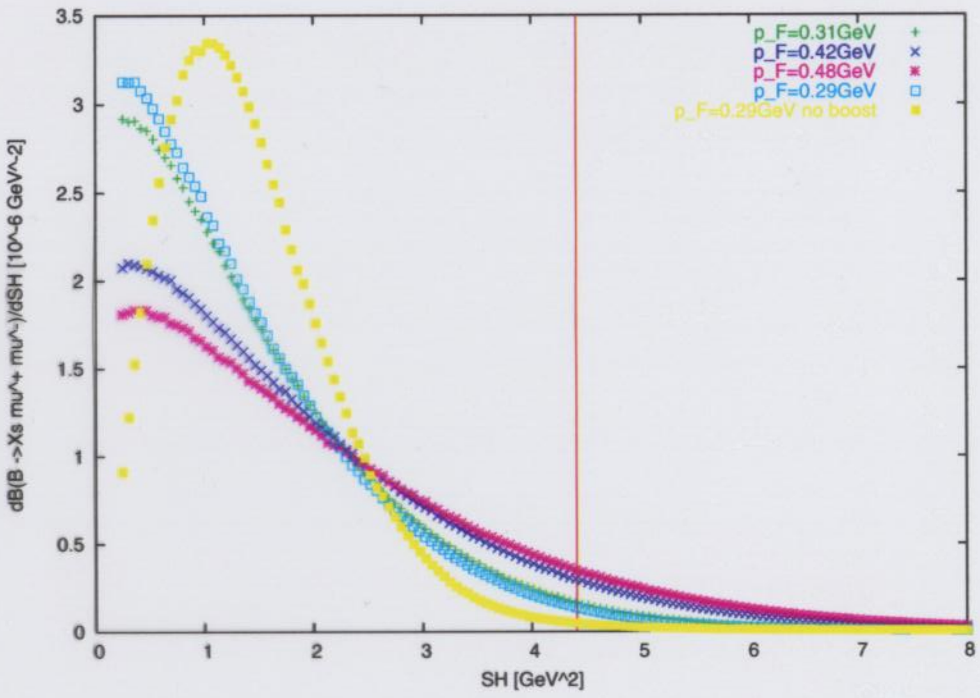
B-factories have $500 fb^{-1}$, $\sigma(stat, sys)_{b\to s\gamma} = 1.8\%, 3\%$ hep-ex/0112041

hadronic invariant mass cuts

Belle'02 $b \to s\ell^+\ell^-$ analysis $m_{X_s} < 2.1$ GeV $\qquad$ m_{X_s}-spectrum:

bremsstrahlung and Fermi motion (b-quark moves in B-meson with p_F)





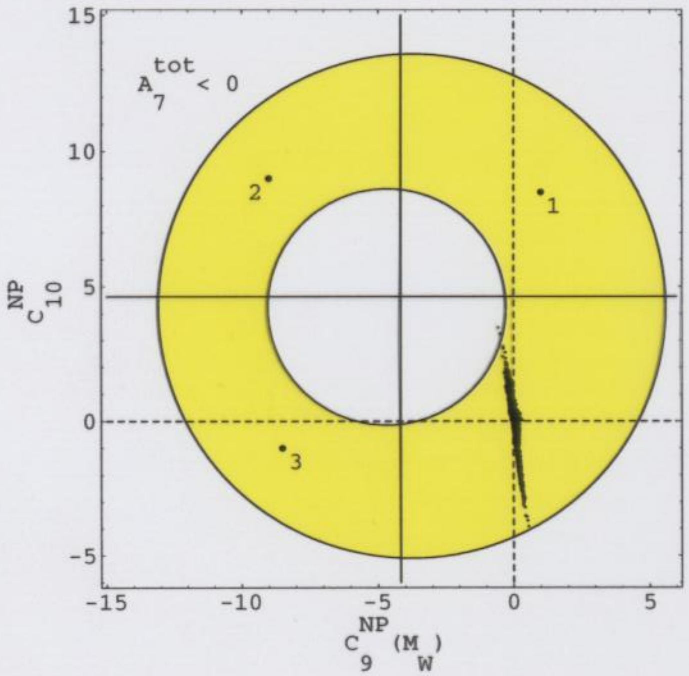
efficiency in Fermi mo $(\int_{m_K}^{2.1GeV} dX_s \frac{dBr}{dX_s})/Br = 93 \pm 4\%$ hep-ph/9807418

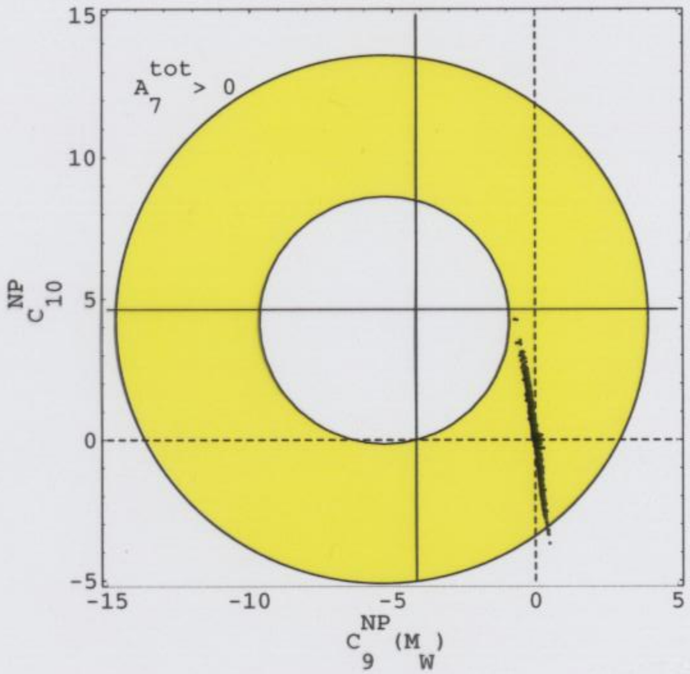
fit $\bar{B} \to X_s\gamma$ photon spectrum by CLEO hep-ex/0108032: $p_F = 410$ MeV

model independent analysis $b \to s\ell^+\ell^-, s\gamma$

SM: $C_9(m_W) = 2.04$, $C_{10} = -4.59$, $C_7 < 0$ hep-ph/0112300

@ 90% C.L. incl data on $B \to K\mu^+\mu^-$,$B \to X_s e^+ e^-$ and $b \to s\gamma$





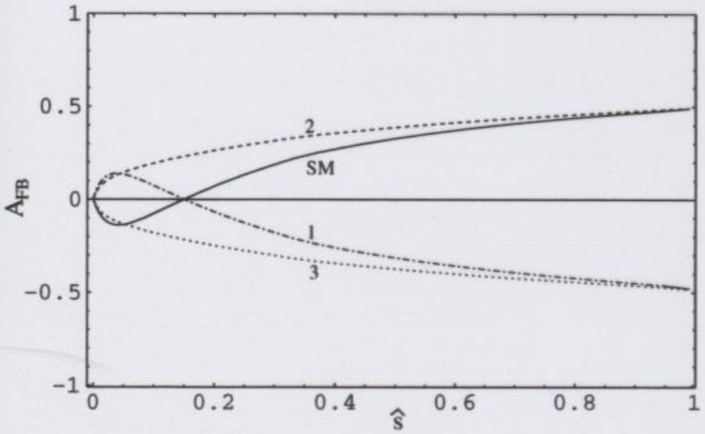
$|C_{10}| \lesssim 2|C_{10}^{SM}|$ bound on sZb-penguins, contrib. also to $b \to s\bar{q}q, b \to s\bar{\nu}\nu$

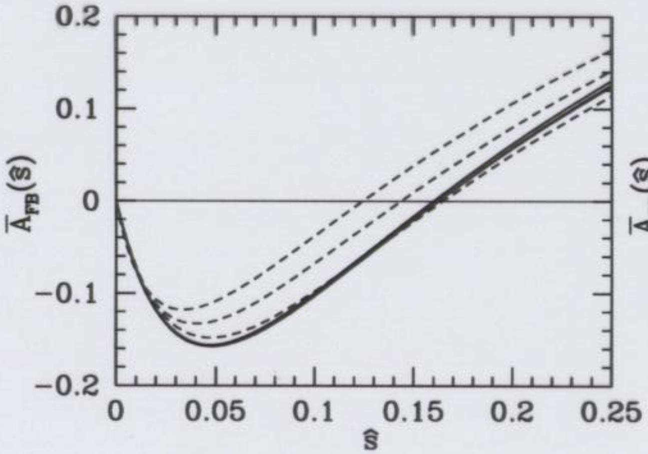
scatter plot: non MFV scenario, with up-squark mixing $\delta^U_{23,LL}, \delta^U_{23,LR}$

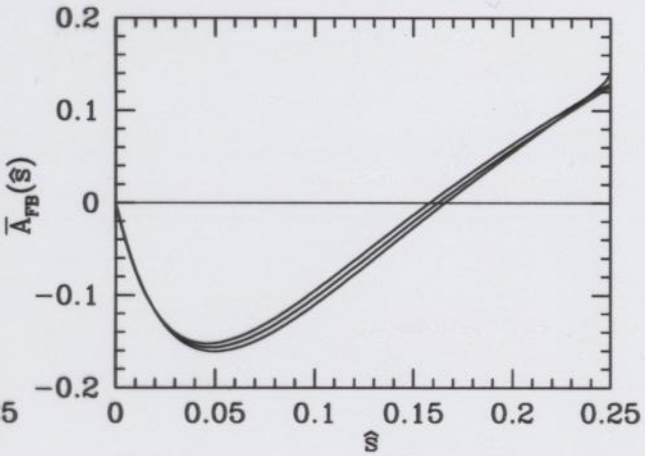
forward backward asymmetry in $b \to s\ell^+\ell^-$

asymmetry between N(forward) and N(backward) scattered ℓ^- in dilepton CMS w.r.t. B-meson $\quad A_{FB}(\hat{s}) \sim C_{10}\left[C_7 + \beta(\hat{s}) Re(C_9)\right]$

- SM or $C_7 < 0$ A_{FB} has zero in low q^2 $\quad \hat{s}_{SM}^{NNLL} = 0.162 \pm 0.002(8)$
- $C_7 > 0$ NO zero (curve 2) e.g. MFV MSSM
- C_{10} non-SM curve 3 or 1 or flat $A_{FB}(\hat{s}) \sim 0$ possible !







mid fig solid NNLO vs dashed NLO for $\mu = m_b/2, m_b, 2m_b, m_c/m_b = 0.29$
right fig NNLO $\mu = m_b$, $m_c/m_b = 0.29 \pm 0.04$ hep-ph/0209006

SUMMARY

- $b \to s \ell^+ \ell^-$ complements $b \to s \gamma$
  (Z-penguins, box)
  test SM PRECISELY @ NNLO
  for low q^2

- FORWARD-BACKWARD ASYMMETRY A_{FB}
  in $b \to s \ell^+ \ell^-$ $\longrightarrow$ TALK by T. HURTH
  in $B \to K^* \ell^+ \ell^-$ $\longrightarrow$ TALK by T. FELDMANN

- theory uncertainties in $b \to s \ell^+ \ell^-$

  * charm — $c\bar{c}$-resonances (factorization ok for $q^2 < 4 m_c^2$)

  normalization (use different one)
  $\longrightarrow$ TALK by M. MISIAK

  * B-boundstate effects — from m_{X_s}-cuts
  (learn from $b \to s \gamma$ E_γ-spectrum)