9. Übungsblatt zur Vorlesung **WS 2016/17**
Quantenfeldtheorie II **Prof. J. Brod**
Abgabe: bis Mi. 18. Januar 2017, 12:00 Uhr **Kasten 233/P2-04-411**

Aufgabe 1: $U(1)$ effective Lagrangian **(8 Punkte)**
Consider a theory with a complex scalar field ϕ with a $U(1)$ global symmetry $\phi \to \phi' = \exp(i\theta)\phi$. The Lagrangian will be

$$\mathscr{L} = \partial_\mu \phi^* \partial^\mu \phi + \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2. \tag{1}$$

- Minimize the potential to find the ground state and write out the Lagrangian in the basis

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \phi_1(x) + i\phi_2(x)). \tag{2}$$

  Show that ϕ_2 is the Goldstone boson.

- Use this Lagrangian to calculate the low-energy scattering of $\phi_2 + \phi_2 \to \phi_2 + \phi_2$. Show that despite the non-derivative interactions interactions of the Lagrangian, cancelations occur such that leading scattering amplitude starts at order p^4.

- Instead of the basis above, express the Lagrangian using an exponential basis,

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \Phi(x)) \exp\big(i\chi(x)/v\big). \tag{3}$$

  Show that in this basis a 'shift symmetry' $\chi \to \chi + c$ is manifest.

- Calculate $\phi_2 + \phi_2 \to \phi_2 + \phi_2$ again in this basis and show that the results agree. Note that the fact that the amplitude is of order p^4 is more readily apparent in this basis.

Aufgabe 2: Symmetries of the linear σ-model **(8 Punkte)**
The Lagrangian for the linear σ-model is given by

$$\begin{aligned}\mathscr{L} =& \frac{1}{2}[(\partial_\mu \pi)^2 + (\partial_\mu \sigma)^2] + \bar{N} i\gamma^\mu \partial_\mu N + g\bar{N}(\sigma + i\gamma_5 \tau \cdot \pi) N \\ &+ \frac{\mu^2}{2}(\sigma^2 + \pi^2) - \frac{\lambda}{4}(\sigma^2 + \pi^2)^2\end{aligned} \tag{4}$$

where $N = (p,\ n)^T$ is an isospin-$\frac{1}{2}$ nucleon field, $\pi = (\pi_1,\ \pi_2,\ \pi_3)^T$ an isospin-one pion field, and σ an isospin-zero scalar field (τ refers the Pauli matrices). It is convenient to use a 2×2 matrix to represent the isospin fields collectively:

$$\Sigma = \sigma + i\tau \cdot \pi. \tag{5}$$

- Show that the Lagrangian is invariant under the isospin transformations: $N \to N' = U N$ and $\Sigma \to \Sigma' = U \Sigma U^\dagger$, where $U = \exp\big(\frac{i}{2}\alpha \cdot \tau\big)$ is an arbitrary 2×2 unitarity matrix with $\alpha = (\alpha_1,\ \alpha_2,\ \alpha_3)$ being a set of real constants. Find the corresponding conserved isospin vector currents V_μ^i, $i = 1, 2, 3$.

- Show that the Lagrangian is invariant under the axial isospin transformations: $N \to N' = U_1\, N$ and $\Sigma \to \Sigma' = U_2^\dagger\, \Sigma\, U_2^\dagger$ where $U_1 = \exp\left(\frac{i}{2}\beta\cdot\tau\gamma_5\right)$ and $U_2 = \exp\left(\frac{i}{2}\beta\cdot\tau\right)$ with $\beta = (\beta_1\ \ \beta_2\ \ \beta_3)$ being a set of real constants. Find the corresponding conserved isospin axial-vector currents A^i_μ, $i = 1,2,3$.

- Calculate the charge commutators

$$[Q^i, Q^j]\ ,\ [Q^i, Q^{5j}]\ ,\ [Q^{5i}, Q^{5j}]\ , \tag{6}$$

  where

$$Q^i = \int d^3x V^i_0(x) \text{ and } Q^{5i} = \int d^3x A^i_0(x) \tag{7}$$

- Calculate the commutators of particle fields with the vector and axial-vector charges

$$[Q^i, N^a]\ ,\ [Q^i, \pi^j]\ ,\ [Q^i, \sigma]\ ,\ [Q^{5i}, N^a]\ ,\ [Q^{5i}, \pi^j]\ ,\ [Q^{5i}, \sigma] \tag{8}$$

  where $a = 1,2$.

Aufgabe 3: Other worlds **(4 Punkte)**

Describe the changes in the macroscopic world if the quark masses were slightly different in the following ways:

- $m_u = m_d = 0$
- $m_u > m_d$
- $m_u = 0,\ m_d = m_s$