12. Übungsblatt zur Vorlesung — WS 2019/20

Einführung in die Elementarteilchentheorie — **Prof. G. Hiller**

Abgabe: bis Di. 14. Januar 2020, 12:00 Uhr — **Kasten 245**

http://people.het.physik.tu-dortmund.de/~ghiller/WS1920ETT.html

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Exercise 1: Drell-Yan processes and New Physics **(8 Points)**

Drell-Yan processes denote lepton production in high energy hadron collisions. An example is electron–positron production in proton–proton collisions at the LHC

$$pp \to e^+e^- + X\,, \tag{1.1}$$

where X denotes any hadronic final state. For large momentum transfer perturbation theory can be used. The total cross section at hadron level σ^{had} can be computed at leading-order QCD as

$$\sigma^{\text{had}}(p(P_1)+p(P_2)\to e^+e^-X) = \int_0^1 dx_1 \int_0^1 dx_2 \sum_f f_f(x_1) f_{\bar f}(x_2)\hat\sigma(q_f(x_1P)+\bar q_f(x_2P)\to e^+e^-)\,, \tag{1.2}$$

where the sum runs over all species of quarks and antiquarks $(u, d, \bar u, \bar d, \ldots)$ and $\hat\sigma$ denotes the partonic cross section. The functions f_i are the parton distribution functions discussed on sheet 9 and x_i are momentum fractions.

Consider in the following that, in addition to the photon γ, a hypothetical *new* massive photon γ' exists. It possesses the same couplings to quarks and leptons as the massless photon γ and its propagator can be parametrised in a Breit-Wigner form, that is $-\mathrm{i}\,g^{\mu\nu}/\left(q^2 - M^2 + \mathrm{i}\,M\Gamma\right)$, where q, M, Γ denote the momentum, mass and decay width of the new photon γ', respectively. Neglect all particle masses in the computation.

a) Draw the parton-level Feynman diagrams at leading order.

b) Calculate the partonic cross section.

c) Calculate the differential hadronic cross section, $\mathrm{d}^2\sigma/\left(\mathrm{d}M_{\ell\ell}^2\mathrm{d}\eta\right)$, with the invariant mass squared $M_{\ell\ell}^2 = q^2$ and the rapidity η of the muon pair, where $q^0 = M_{\ell\ell}\cosh\eta$. How does the $M_{\ell\ell}^2$–spectrum for the differential cross section change (qualitatively) in the case of the new photon?
Hint: Express $M_{\ell\ell}^2$ and η in terms of x_1 and x_2.

d) In analogy to Drell-Yan processes of pp–scattering also $p\overline{p}$-processes exist. Explain the differences in the calculation of the hadronic cross sections for $p\overline{p}$– and pp–scattering.

Exercise 2: Discrete symmetries **(6 Points)**

The operators for parity P, charge conjugation C and time reversal T each constitute a discrete group with the identity $\mathbb{1}$ containing only two elements, which can be seen by the relation $P^2 = C^2 = T^2 = \mathbb{1}$. These operators play a vital role in particle physics, e.g. the combination of operators CPT is a symmetry of the QFT. The effect of C, P, T on Dirac fields are given by

$$\begin{aligned} C\,\psi\,(t,\vec{x})\;C &= -\mathrm{i}\zeta\,\left(\overline{\psi}\,(t,\vec{x})\,\gamma^0\gamma^2\right)^T\,, \\ P\,\psi\,(t,\vec{x})\;P &= \eta\gamma^0\,\psi\,(t,-\vec{x})\,, \\ T\,\psi\,(t,\vec{x})\;T &= \xi\,\gamma^1\gamma^3\,\psi\,(-t,\vec{x})\,, \end{aligned} \tag{2.1}$$

where ζ, η, ξ denote complex phases, which can be set to 1 for the purpose of this exercise. The operators P and C are linear and unitary, whereas T is anti-linear and anti-unitary, which translates to

$$\begin{aligned} T\,\left(a\psi_1 + b\psi_2\right) &= a^*\,T\,\psi_1 + b^*\,T\,\psi_2\,, \\ \left\langle T\,\phi_1\middle|T\,\phi_2\right\rangle &= \left\langle\phi_1\middle|\phi_2\right\rangle^*\,, \end{aligned} \tag{2.2}$$

with states $|\phi_{1,2}\rangle$, operators $\psi_{1,2}$ and complex numbers a, b. [1]

	$\overline{\psi}\psi$	$\mathrm{i}\overline{\psi}\gamma^5\psi$	$\overline{\psi}\gamma^\mu\psi$	$\overline{\psi}\gamma^\mu\gamma^5\psi$	$\overline{\psi}\sigma^{\mu\nu}\psi$
P	$+1$	-1	$(-1)^\mu$	$-(-1)^\mu$	$(-1)^\mu(-1)^\nu$
T	$+1$	-1	$(-1)^\mu$	$(-1)^\mu$	$-(-1)^\mu(-1)^\nu$
C	$+1$	$+1$	-1	$+1$	-1
CPT	$+1$	$+1$	-1	-1	$+1$

a) In the table above a list of transformation properties under the discrete symmetries is given. Show that the bilinear forms of bispinors fulfill these properties for the first three columns. (Keep in mind that space-time arguments of the bispinors might not necessarily stay invariant.)
The notation

$$(-1)^\mu \equiv \begin{cases} +1 & \text{for} \quad \mu = 0 \\ -1 & \text{for} \quad \mu = 1,2,3 \end{cases} \tag{2.3}$$

is understood.

b) What is the effect of P and T on 4-vectors $x = (t, \vec{x})$?
Use this to show the invariance of the free Lagrangian

$$\mathscr{L} = \overline{\psi}(x)\,(\mathrm{i}\partial\!\!\!/ - m)\,\psi(x) \tag{2.4}$$

under P and T.

c) Why does the last row of the table imply that all possible Lorentz-invariant Lagrangians with Dirac structure are also invariant under CPT transformation?

Exercise 3: Fierz identities **(6 Points)**

The Fermi theory of weak interaction includes (Lorentz-invariant) contractions of 4-fermion fields $\left(\overline{\psi}_1(x)\,\Gamma^A\,\psi_2(x)\right)\left(\overline{\psi}_3(x)\,\Gamma^B\,\psi_4(x)\right) \subset \mathscr{L}$. ψ_i denote the fermion fields and

[1] A derivation of these features is presented in e.g. Peskin Schroeder, ch. 3.6. The anti-unitarity and anti-linearity of T is due to the fact that otherwise states with arbitrary negative energy eigenvalues are included in the theory.

the possible Dirac structures Γ^l are given in Eq. (3.1).
Does the reorder of the fermion fields induce additional terms that modify the Lagrangian? To answer this question we introduce *Fierz* identities[2].

We define a chiral basis

$$\Gamma^A \in \{P_R, P_L, \gamma^\mu P_L, \gamma^\mu P_R, \sigma^{\mu\nu}\}\,, \tag{3.1}$$

and the associated dual basis

$$\Gamma_A \in \left\{P_R, P_L, \gamma_\mu P_R, \gamma_\mu P_L, \frac{\sigma_{\mu\nu}}{2}\right\}, \tag{3.2}$$

where $P_{L/R} = \frac{(1\mp\gamma_5)}{2}$ and $\sigma^{\mu\nu} = \frac{\mathrm{i}}{2}[\gamma^\mu, \gamma^\nu]$. Notice the order of elements in Eqs. (3.1) and (3.2). The Fierz identity is then given by

$$\left(\overline{\psi}_1\,\Gamma^A\,\psi_2\right)\left(\overline{\psi}_3\,\Gamma^B\,\psi_4\right) = -\frac{1}{4}\,\mathrm{Tr}\left[\Gamma^A\Gamma_D\Gamma^B\Gamma_C\right]\left(\overline{\psi}_1\,\Gamma^C\,\psi_4\right)\left(\overline{\psi}_3\,\Gamma^D\,\psi_2\right). \tag{3.3}$$

a) First, consider the general structure of the Fierz identity. Why is $\mu < \nu$ sufficient in Eqs. (3.1) and (3.2)? Explain the minus-sign on the right-hand side of Eq. (3.3). Finally, show that the normalisation $\mathrm{Tr}\left(\Gamma^A\,\Gamma_B\right) = 2\,\delta^A_B$ holds for $\Gamma^A = P_R$.

b) Now calculate the contraction

$$\left(\overline{\psi}_1\,P_R\,\psi_2\right)\left(\overline{\psi}_3\,P_L\,\psi_4\right). \tag{3.4}$$

[2] A mathematical formulation is presented in e.g. C.C. Nishi, *Simple derivation of general Fierz-like identities*, Am.J.Phys. **73**, 1160 (2005) hep-ph/0412245.