10. Übungsblatt zur Vorlesung — WS 2019/20

Einführung in die Elementarteilchentheorie — **Prof. G. Hiller**

Abgabe: bis Di. 17. Dezember 2019, 12:00 Uhr — **Kasten 245**

http://people.het.physik.tu-dortmund.de/~ghiller/WS1920ETT.html

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NOTE: This is a link for the evaluation of ETT19/20, the password will be given in the next lecture.

https://evaluation.tu-dortmund.de/evasys/online.php

Exercise 1: Cross section for $q\bar{q} \to Q\bar{Q}$ scattering **(10 Points)**

Consider the process $q\bar{q} \to Q\bar{Q}$ with different quarks q and Q in the initial and final state.

a) Draw all Feynman diagrams that contribute to the process to leading order.

b) Calculate the matrix element $\mathcal{M}_{abcd}$ and show that it factorizes in the following way

$$\mathcal{M}_{abcd} = \mathcal{M}_{\text{Dirac}}\,\Phi_{abcd}\,, \tag{1.1}$$

where

$$|\mathcal{M}_{abcd}|^2 = |\mathcal{M}_{\text{Dirac}}|^2\,|\Phi_{abcd}|^2\,. \tag{1.2}$$

Φ_{abcd} is the color factor of QCD, whereas $\mathcal{M}_{\text{Dirac}}$ contains the Dirac structure of the matrix element. The color factor can always be factorized in this way.

Hint: For the calculation of the matrix element use the following representation of the QCD Feynman rules:

incoming particle:	$q_i(p)$	$\to$	$u(p)e_i$
outgoing particle:	$q_i(p)$	$\to$	$\bar{u}(p)e_i^\dagger$
incoming anti particle:	$\overline{q}_i(p)$	$\to$	$\bar{v}(p)e_i^\dagger$
outgoing particle:	$\overline{q}_i(p)$	$\to$	$v(p)e_i$
quark gluon vertex:	Γ_μ	$\to$	$\mathrm{i}g_s\gamma_\mu\lambda_A$
gluon propagator in Feynman gauge:	$D_F(q^2)$	$\to$	$\dfrac{-\mathrm{i}g^{\mu\nu}\delta^{AB}}{q^2}$

small latin indices are summed over the three unit vector e_i of the **fundamental** representations of $SU(3)$ (**3** and $\overline{\mathbf{3}}$), i.e. these are the color indices of quarks. Capital latin indices are running over the **adjoint** representation of $SU(3)$ (**8**), which is built by the generators λ_A. Greek indices are minkowski indices, as usual. The usual sum convention holds true for all of these indices.

c) Calculate $\overline{|\mathcal{M}|}^2$. Average over spins and color in the initial state and sum in the final state. For your calculation you can use the result

$$\overline{|\mathcal{M}_{\mathrm{Dirac}}|}^2 = g_s^4\,(1+\cos^2\theta)\,, \tag{1.3}$$

where θ is the scattering angle between the incoming quark q and the outgoing quark Q. Explain why you can use this result and where it comes from.

Hint: Use the Feynman trace technique for the $SU(3)$ structure . Use the identity

$$\{\lambda_A, \lambda_B\} = \frac{1}{3}\,\delta_{AB} + d_{ABC}\,\lambda_C\,. \tag{1.4}$$

The symmetric structure constants d_{ABC} are not necessary for your calculation. Explain why!