8. Übungsblatt zur Vorlesung — WS 2019/20

Einführung in die Elementarteilchentheorie — **Prof. G. Hiller**

Abgabe: bis Di. 03. Dezember 2019, 12:00 Uhr — **Kasten 245**

http://people.het.physik.tu-dortmund.de/~ghiller/WS1920ETT.html

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Exercise 1: The rare decay $\overline{B}_s \to \mu^+\mu^-$ – Part I **(8 Points)**

Introductory remarks

The quark level transition $b \to s\mu^+\mu^-$ is a very important test of the Standard Model and is responsible for the following processes

- $\overline{B} \to \overline{X}_s\mu^+\mu^-$ (inclusive semileptonic decay),
- $\overline{B} \to \overline{K}^{(*)}\mu^+\mu^-$ (exclusive semileptonic decay),
- $\overline{B}_s \to \mu^+\mu^-$ (leptonic decay).

Sheet 8 will investigate the process $\overline{B}_s \to \mu^+\mu^-$ in more detail. The Standard Model prediction for the branching ratio of this decay is (arxiv/1311.0903)

$$\mathscr{B}^{\text{SM}}(\overline{B}_s \to \mu^+\mu^-) = (3.65 \pm 0.23) \cdot 10^{-9}\,, \tag{1.1}$$

i.e. of about $3\cdot 10^8$ produced $\overline{B}_s$ mesons, statistically only one decays into a pair of muons. The decay rate may be different, if physics beyond the Standard Model (BSM) exists and contributes to the quark level transition.

On the experimental side the decay was first seen in a combined analysis of CMS and LHCb data (arxiv/1411.4413), updated in (arxiv/1703.05747) and the current world average can be found in the PDG[1]

$$\mathscr{B}^{\text{exp}}(\overline{B}_s \to \mu^+\mu^-) = (3.0 \pm 0.4) \cdot 10^{-9}\,, \tag{1.2}$$

in agreement with the prediction in the Standard Model. Note that in the common PDG convention a $\overline{B}_s$ meson is a bound state containing a b quark and an anti s quark. A B_s meson is then a bound state formed by an anti b and an s quark.

Exercises

(a) The decay $\overline{B}_s \to \mu^+\mu^-$ corresponds to an annihilation of a b quark with an anti s quark, plus the creation of a $\mu^+\mu^-$-pair. The first process is described by a matrix element $\langle 0|\overline{\psi}_s\Gamma\psi_b|\overline{B}_s(p)\rangle$. Here p denotes the four-momentum of the B meson and Γ a matrix contained in the space spanned by the Dirac structures $\Gamma_a = \{1, \gamma_5, \gamma^\mu, \gamma^\mu\gamma_5, \sigma^{\mu\nu}\}$. The Fourier decompositions of ψ_s, ψ_b are the same as the ones in Eqs. (2.5) and (2.6) on sheet 4, but the creation and annihilation operators additionally carry the flavor indices b, s for the b and s quark, respectively.

Why does $\langle 0|\overline{\psi}_s\Gamma\psi_b|B_s(p)\rangle$ vanish and how should a matrix element for the annihilation of a B_s meson look like?

(b) We now try to decompose the matrix element in (a) $\langle 0|\overline{\psi}_s\Gamma\psi_b|\overline{B}_s(p)\rangle$ into lorentz invariants. p^μ is the only available four-vector. The so called decay constant f_{B_s} of

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[1] Particle data group - very important reference for phenomenologists, contains all particle properties.

the $\overline{B}_s$ is defined as the constant of proportionality for the axial-vector compontent, i.e.

$$\langle 0|\overline{\psi}_s\gamma^\mu\gamma_5\psi_b|\overline{B}_s(p)\rangle = -\mathrm{i}\, f_{B_s}\, p^\mu\,. \tag{1.3}$$

Why does the same matrix element with the vector current $\langle 0|\overline{\psi}_s\gamma^\mu\psi_b|\overline{B}_s(p)\rangle$ vanish?

HINT: Use that the strong interaction, responsible for the bound state, conserves parity, and B_s mesons are pseudo scalar particles.

(c) Why does $\langle 0|\overline{\psi}_s\gamma^\mu\gamma^\nu\psi_b|\overline{B}_s(p)\rangle$ vanish?

Exercise 2: The rare decay $\overline{B}_s \to \mu^+\mu^-$ – Part II **(12 Points)**

Introductory remarks

The quark transition $b \to s\mu^+\mu^-$ is described at low energies by an interaction lagrangian density of the form

$$\mathcal{L}^{bs}_{\text{int}} = G_F\, C_A[\overline{\psi}_s\gamma^\mu(1-\gamma_5)\psi_b] \times [\overline{\psi}_\mu\gamma_\mu\gamma_5\psi_\mu] + h.c. \tag{2.1}$$

Here, the value of the effective coupling constant C_A follows from loop calculations in the Standard Model and is given by

$$C_A^{\text{SM}} = |V_{ts}^* V_{tb}|\,\frac{\alpha_{em}(m_W)}{\sqrt{8}\pi\sin^2\theta_W}\, Y_{\text{SM}}\left(\frac{m_t^2}{m_W^2}\right), \tag{2.2}$$

with $Y_{\text{SM}}(m_t^2/m_W^2) = 1.044$ and $\alpha_{em}(m_W) \approx 1/128$. All other constants are given in the table on the next page.

Exercises

(a) Calculate the decay amplitude of the decay $\overline{B}_s \to \mu^+\mu^-$

$$\mathcal{A}(\overline{B}_s \to \mu^+\mu^-) = \mathrm{i}\,\langle\mu^+\mu^-|\overline{\psi}_\mu\gamma_\mu\gamma_5\psi_\mu|0\rangle \cdot \langle 0|\overline{\psi}_s\gamma^\mu(1-\gamma_5)\psi_b|\overline{B}_s\rangle\,. \tag{2.3}$$

Use the definition of the decay constant from the first exercise! Why do the two currents factorize as given in Eq. (2.3)?

(b) Show that after summing the muon spins the decay rate is given by

$$\Gamma(\overline{B}_s \to \mu^+\mu^-) = \frac{G_F^2 f_{B_s}^2\, m_{B_s}\, m_\mu^2}{2\pi}\sqrt{1-\frac{4m_\mu^2}{m_{B_s}^2}}\,|C_A|^2\,. \tag{2.4}$$

(c) Plot the decay rate versus the muon mass in a range of $m_\mu \in [0,3]$ GeV. Discuss the plot and calculate the values for the electron mass m_e and the tau mass m_τ.

(d) *NEW PHYSICS* can contribute to the decay by altering the value of the effective coupling constant $C_A = C_A^{\text{SM}} + C_A^{\text{NP}}$. Use whatever you find on this sheet to give an upper bound for additional contributions, i.e. calculate:

$$\frac{|C_A^{\text{NP}}|}{|C_A^{\text{SM}}|} < \mathbf{?} \tag{2.5}$$

HINTS:

For the decay rate we find in the PDG:

$$\Gamma(A \to B_1 B_2) = \frac{|\vec{k}_1|}{8\pi\, m_A^2}\sum_{\text{spins}}|\mathcal{A}|^2 \tag{2.6}$$

More information for sheet 8:

constant	value	explanation
G_F	$1.166364 \cdot 10^{-5}\,\text{GeV}^{-2}$	fermi coupling constant
$\hbar$	$6.58211899 \cdot 10^{-22}\,\text{MeV s}$	reduced Planck constant
m_e	$0.511\,\text{MeV}$	electron mass
m_μ	$105.658\,\text{MeV}$	muon mass
m_τ	$1777\,\text{MeV}$	tau mass
f_{B_s}	$231\,\text{MeV}$	$\overline{B}_s$ decay constant
m_{B_s}	$5.366\,\text{GeV}$	$\overline{B}_s$ mass
$\sin^2\theta_W$	0.23116	Weinberg angle
$\lvert V_{ts}^* V_{tb} \rvert$	0.0403	CKM matirx elements
$\tau_{\overline{B}_s}$	$142.5 \cdot 10^{-14}\,\text{s}$	$\overline{B}_s$ lifetime