7. Übungsblatt zur Vorlesung — WS 2019/20

Einführung in die Elementarteilchentheorie — **Prof. G. Hiller**

Abgabe: bis Di. 26. November 2019, 12:00 Uhr — **Kasten 245**

http://people.het.physik.tu-dortmund.de/~ghiller/WS1920ETT.html

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Exercise 1: Bhabha scattering **(12 Points)**

Compute the differential cross section $d\sigma/d\cos\theta$ for the process $e^+e^- \to e^+e^-$ using the QED Feynman rules at leading order. Here, θ denotes the scattering angle in the center of mass frame of the incoming fermions. The electron mass can be neglected in this calculation.

(a) Draw all lowest order QED Feynman diagrams for this process.

(b) Give the corresponding matrix elements M_i for each diagram and the total matrix element $M = \sum_i M_i$.

(c) Use your results from (b) to compute the spin-averaged squared matrix element

$$\overline{|M|}^2 = \frac{1}{(2S_1+1)(2S_2+1)} \sum_{Spins} |M|^2 , \tag{1.1}$$

where S_1 and S_2 denote the spins of the initial state particles.

(d) Show that the differential cross section is given as

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{s}\left[\frac{u^2+t^2}{s^2} + \frac{2u^2}{st} + \frac{u^2+s^2}{t^2}\right], \tag{1.2}$$

where s, t and u are the Mandelstam variables and $\alpha = e^2/(4\pi)$.

(e) Why and how can you relate the result from (d) to the differential cross section for Møller scattering $e^-e^- \to e^-e^-$

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{s}\left[\frac{s^2+t^2}{u^2} + \frac{2s^2}{ut} + \frac{u^2+s^2}{t^2}\right]? \tag{1.3}$$

Exercise 2: $e^+e^- \to \pi^+\pi^-$ **pion production** **(8 Points)**

Consider the following reaction:

$$e^+(k_+)e^-(k_-) \to \pi^+(p_+)\pi^-(p_-)\,, \tag{2.1}$$

in the center-of-mass frame of the leptons. You can neglect the masses of the leptons. The pions $\pi^\pm$ are hadrons with spin 0 and charge ± 1e . The mass of the pion is $m_{\pi^\pm} =$ 139.57 MeV. The matrix element of the hadronic current reads

$$J_\mu^{\pi^+\pi^-} = a(q^2)(p_+ - p_-)_\mu + b(q^2)(p_+ + p_-)_\mu\,, \tag{2.2}$$

where $a(q^2)$ and $b(q^2)$ are formfactors and $q^2 = (p_+ + p_-)^2$.

(a) Draw the lowest order QED Feynman diagram for the process in Eq. (2.1). Why is Eq. (2.2) already the complete expression for the hadronic current, that is, why are there no other terms in this equation?

(b) Imposing current conservation $q^\mu J_\mu^{\pi^+\pi^-} = 0$, what does this mean for the form factors?

(c) The Feynman rule for the photon coupling to pions reads $+\mathrm{i}eJ_\mu^{\pi^+\pi^-}$. The rule for external scalars is simply 1. Determine the matrix element for the process in Eq. (2.1) up to leading order.

(d) Calculate the differential cross section $\mathrm{d}\sigma/\mathrm{d}\cos\theta$, where θ denotes the scattering angle between e^- and π^-. Calculate the total cross section in the high energy limit.