2. Übungsblatt zur Vorlesung — WS 2019/20

Einführung in die Elementarteilchentheorie — **Prof. G. Hiller**

Abgabe: bis Di. 22. Oktober 2019, 12:00 Uhr — **Kasten 245**

http://people.het.physik.tu-dortmund.de/~ghiller/WS1920ETT.html

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Exercise 1: Optical theorem **(5 Points)**

Check the validity of the optical theorem in the special case of the scattering of two spinless particles interacting with each other via a central potential $V(r) = V(|r|)$. The scattering cross section can be represented as a sum over partial waves:

$$\frac{d\sigma}{d\Omega} = \left|f(k,\theta)\right|^2 , \tag{1}$$

$$f(k,\theta) = \frac{1}{k}\sum_{l=0}^{\infty}(2l+1)\left(\frac{e^{2i\delta_l}-1}{2i}\right)P_l(\cos\theta) , \tag{2}$$

where

- k is the absolute value of the particle's momentum in the center of mass system,
- θ is the scattering angle,
- $P_l(x)$ is the l-th Legendre polynomial (due to the azimuthal symmetry only the waves with L_z eigenvalues $m = 0$ contribute),
- $\delta_l = \delta_l(k)$ is the scattering phase shift.

The optical theorem is:

$$\sigma_{\text{tot}}(k) = \frac{4\pi}{k}\text{Im} f(k,\theta = 0) . \tag{3}$$

Exercise 2: Lorentz invariance violation **(5 Points)**

In this exercise, Lorentz invariance violating contributions are examined. Such contributions could for example stem from a yet unknown quantum gravity theory due to a discrete space-time structure, as assumed in some models at the Planck scale $M = 10^{19}$ GeV. Violation of Lorentz invariance could lead to modified dispersion relations. In the case of the photon such a relation could take the following form in the lowest order:

$$E_\gamma^2 = k^2 + \epsilon k^2 + \frac{\xi}{M}k^3 , \tag{4}$$

where k denotes the absolute value of the three momentum of the photon, E_γ is the photon energy and ϵ, ξ are dimensionless parameters.

(a) What are the conventional vacuum dispersion relations for photons or electrons and positrons? Show that the altered relation in Eq. (4) violates Lorentz invariance.

(b) Compute the photon group velocity using the dispersion relation in Eq. (4) up to first order in k/M.

(c) Consider the decay of a photon in an electron-positron-pair, $\gamma \to e^+ e^-$. Examine the case $\epsilon > 0$, $\xi < 0$. Start from energy conservation and show, under the assumption $|\vec{p}_+| = |\vec{p}_-|$, the relation

$$\epsilon k^2 + \frac{\xi}{M} k^3 \geq 4 m_e^2 , \tag{5}$$

where m_e denotes the electron mass.

(d) In the case $\xi = -1$, for which value of ϵ is the decay possible?

(e) Show that in general (as long as the decay is possible) there exists not only a lower bound on k, but also an upper bound, so that the decay is only possible in a certain momentum range.

Exercise 3: Phase space three body decay **(10 Points)**

(a) Simplify the phase space integral of a matrix element $\mathcal{M}$ for a three body decay $a \to 1+2+3$:

$$R_3(|\mathcal{M}|^2) = \int\int\int \frac{\mathrm{d}^3 p_1}{2E_1} \frac{\mathrm{d}^3 p_2}{2E_2} \frac{\mathrm{d}^3 p_3}{2E_3} \delta^4(p_a^\mu - p_1^\mu - p_2^\mu - p_3^\mu) |\mathcal{M}|^2 . \tag{6}$$

Instruction:

- Use the relation

$$\delta(f(x)) = \sum_i \frac{1}{|f'(x_{0i})|} \delta(x - x_{0i}) , \tag{7}$$

which holds for functions f with finitely many simple roots x_{0i}, to show that

$$\int \frac{\mathrm{d}^3 p}{2E_p} = \int \mathrm{d}^4 p \delta((p^0)^2 - |\vec{p}|^2 - m^2) \Theta(p^0) . \tag{8}$$

- Use equation (8) to perform the $\mathrm{d}^3 p_3$-integration in the rest frame of particle a.
- Simplify the δ-function. The final result should have the form

$$R_3(|\mathcal{M}|^2) = \int\int \frac{\mathrm{d}^3 p_1^*}{2E_1} \frac{\mathrm{d}^3 p_2^*}{2E_2} \theta(p^0) \delta(p^\mu p_\mu - m_3^2) |\mathcal{M}|^2 \tag{9}$$

with $p^\mu = p_a^\mu - p_1^\mu - p_2^\mu$. The star $(^*)$ denotes variables in the rest frame of particle a.

(b) Now consider the decay $\mu^- \to e^- \nu_\mu \bar{\nu}_e$. You can neglect the masses of the decay products in the following. Decompose the differentials as $\frac{\mathrm{d}^3 p}{E_p} = E_p \mathrm{d}E_p \mathrm{d}\Omega$. Figure out the relative position of the $\vec{p}_i^*$ with respect to each other in order to perform the solid angle integration.